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Study of K- Hsu Structure in Generalized Almost Contact Manifold

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Abstract: Cartesian product of two manifolds has been defined and studied by Pandey[2]. In this paper we have taken Cartesian product of k-Hsu-Structure manifolds, where k is some finite integer, and studied some properties of curvature and Ricci tensor of such a product manifold.

Key words & Phases: k-Hsu-Structure manifolds, generalized almost contact structure, KH-structure.

1. Introduction

Let $M_1, M_2, ..., M_k$ be k-Hsu-structure manifolds each of class C^∞ and of dimension $n_1, n_2, ..., n_k$ respectively. Suppose $(M_1)m_1, (M_2)m_2, ..., (M_k)m_k$, be their tangent spaces at $m_1 \in M_1, m_2 \in M_2, ..., m_k \in M_k$, then the product space $(M_1)m_1 \times (M_2)m_2 \times .... \times (M_k)m_k$, contains vector fields of the form $(X_1, X_2, ..., X_k)$, where $X_1 \in (M_1)m_1, X_2 \in (M_2)m_2, ..., X_k \in (M_k)m_k$. Vector addition and scalar multiplication on above product space are defined as follows:

(1.1) $(X_1, X_2, ..., X_k) + (Y_1, Y_2, ..., Y_k) = (X_1 + Y_1, X_2 + Y_2, ..., X_k + Y_k)$

(1.2) $\lambda(X_1, X_2, ..., X_k) = (\lambda X_1, \lambda X_2, ..., \lambda X_k)$,

where $X_i, Y_i \in (M_i)m_i$, $i = 1, 2, ..., k$ and λ is a scalar.

Under these conditions the product space $(M_1)m_1 \times (M_2)m_2 \times .... \times (M_k)m_k$ forms a vector space.

A linear transformation F on the product space is defined as

(1.3) $F(X_1, X_2, ..., X_k) = (F_1 X_1, F_2 X_2, ..., F_k X_k)$,

where $F_1, F_2, ..., F_k$ are linear transformations on $(M_1)m_1, (M_2)m_2, ..., (M_k)m_k$ respectively.

If $f_1, f_2, ..., f_k$ be C^∞ functions over the spaces $(M_1)m_1, (M_2)m_2, ..., (M_k)m_k$ respectively, we define the

C^{∞} function $f_1, f_2, ..., f_k$ on the product space as

(1.4) $(X_1, X_2, ..., X_k)(f_1, f_2, ..., f_k) = (X_1 f_1, X_2 f_2, ..., X_k f_k).$

Let $D_1, D_2, ..., D_k$ be the connections on the manifolds $M_1, M_2, ..., M_k$ respectively. We define the operator D on the product space as

(1.5) $D(X_1, X_2, ..., X_k)(Y_1, Y_2, ..., Y_k) = (D_{1X_1}Y_1, D_{2X_2}Y_2, ..., D_{kX_k}Y_k).$

Then D satisfies all four properties of a connection and thus it is a connection on the product manifold.

2. Some Results

Definition: Let there be defined on V_n, a vector valued linear function F of class C such that

$$F^2 = a^r I_n \qquad 0 \le r \le n$$

where r is an integer and a is real or imaginary number. Then F is called Hsu-structure and V_n is called the **Hsu-structure manifold.**

Theorem 2.1: The product manifold $M_1 \times M_2 \times .... \times M_k$ admits a Hsu-structure if and only if the manifolds $M_1, M_2, ..., M_k$ are Hsu-structure manifolds.

Proof: Suppose $M_1, M_2, ..., M_k$ are Hsu-structure manifolds. Thus there exist tensor fields $F_1, F_2, ..., F_k$ each of type (1, 1) on $M_1, M_2, ..., M_k$ respectively satisfying

(2.1) $F^2 i(X_i) = a^r X_{i}, _n \qquad i = 1, 2, ..., k$

where a is any complex number, not equal to zero.

In view of equation (1.3) it follows that there exists a linear transformation F on $M_1 \times M_2 \times .... \times M_k$ ssatisfying

(2.2) $F^2 i(X_1, X_2, ..., X_k) = (F_1^{\ 2} X_1, F_2^{\ 2} X_2, ..., F_k^{\ 2} X_k)$

$$= a^r (X_1, X_2, ..., X_k)$$

Thus, the product manifold admits a Hsu-structure.

Let us define a Riemannian metric g on the product manifold $M_1 \times M_2 \times .... \times M_k$ as

(2.3) $a^r g((X_1, X_2, ..., X_k), (Y_1, Y_2, ..., Y_k)) = a^r g_1(X_1, Y_1) + a^r g_2(X_2, Y_2) + ... + a^r g_k(X_k, Y_k)$

where $g_1, g_2, ..., g_k$ are the Riemannian metrics over the manifolds $M_1 \times M_2 \times .... \times M_k$ respectively.

If $\xi_1, \xi_2, ..., \xi_k$ be vector fields and $\eta_1, \eta_2, ..., \eta_k$ be 1-forms on the Hsu-structure manifolds

$M_1, M_2, ..., M_k$ respectively, then a vector field ξ and a 1-form η on the product manifold $M_1, M_2, ..., M_k$ is defined.

We now prove the following results.

Theorem 2.2: The product manifold $M_1 \times M_2 \times .... \times M_k$ admits generalized almost contact structure if and only if the manifolds $M_1, M_2, ..., M_k$ possess the same structure.

Proof: Let $M_1, M_2, ..., M_k$ are generalized almost contact manifolds. Thus there exists tensor fields F_i of type (1, 1) vector fields ξ_i and 1-form. $\eta_i, i = 1, 2, ..., k$ satisfying

(2.4) $F_i^2(X_i) = a^r X_i + \eta_i(X_i)\xi_i$

For product manifold $M_1 \times M_2 \times .... \times M_k$.

$$F^2(X_1, X_2, ..., X_k) = (F_1^2 X_1, F_2^2 X_2, ..., F_k^2 X_k)$$

By the help of equation (2.4), takes the form

$$F^2(X_1, X_2, ..., X_k) = a^r(X_1, X_2, ..., X_k) + (\eta_1(X_1)\xi_1, \eta_2(X_2)\xi_2, ..., \eta_k(X_k)\xi_k),$$

or

(2.5) $F^2(X) = a^r X + \eta(X)\xi$.

Hence the product manifold admits a generalized almost contact structure.

Theorem 2.3: The product manifold $M_1 \times M_2 \times .... \times M_k$ admits a KH-structure if and only if the manifolds $M_1, M_2, ..., M_k$ are KH-structure manifolds.

Proof: Suppose $M_1, M_2, ..., M_k$ are KH-structure manifolds. Thus

$$(D_{1X_1}F_1)(Y_1) = (D_{2X_2}F_2)(Y_2)$$

(2.6)
$$= ....$$
$$= ....$$
$$= (D_{kX_k}F_k)(Y_k)$$
$$= 0$$

As D is a connection on the product manifold, we have

(2.7) $(D_{(X_1, X_2, ..., X_k)}F)(Y_1, Y_2, ..., Y_k) = D_{(X_1, X_2, ..., X_k)}\{F(Y_1, Y_2, ..., Y_k)\}$

$$- F\{D_{(X_1, X_2, ..., X_k)}(Y_1, Y_2, ..., Y_k)\}$$

In view of equation (1.3) and equation (1.5), this takes the form

$$(D_{(X_1,X_2,....,X_k)}F)\,(Y_1,Y_2,....,Y_k)=D_{(X_1,X_2,....,X_k)}\,(F_1Y_1,F_2Y_2,....,F_kY_k)$$

$$-F\,(D_{1X_1}Y_1,D_{2X_2}Y_2,....,D_{kX_k}Y_k)$$

$$=-(D_{1X_1}F_1Y_1,D_{2X_2}F_2Y_2,....,D_{kX_k}F_kY_k)$$

$$-(F_1D_{1X_1}Y_1,F_2D_{2X_2}Y_2,....,F_kD_{kX_k}Y_k)$$

$$=((D_{1X_1}F_1)(Y_1),(D_{2X_2}F_2)(Y_2),....,(D_{kX_k}F_k)(Y_k)$$

$$=0.$$

Thus, the product manifold is KH-structure manifold.

Theorem 2.4: The product manifold $M_1\times M_2\times....\times M_k$ of Hsu-structure manifolds $M_1, M_2, ..., M_k$ is almost Tachibana if and only if the manifolds $M_1, M_2, ..., M_k$ are separately Tachibana manifolds.

Proof: Let a Hsu-structure manifolds $M_1, M_2, ..., M_k$ are almost Tachibana manifolds. Then

(2.8) $(D_{iX_i}F_i)(Y_i)+(D_{iY_i}F_i)(Y_i)=0, \qquad i=1,2,....,k.$

3. Curvature and Ricci Tensor

Let $X=(X_1, X_2, ..., X_k)$ and $Y=(Y_1, Y_2, ..., Y_k)$ be C^∞ vector fields on the product manifold $M_1\times M_2\times....\times M_k$ and $F=(f_1, f_2, ..., f_k)$ be a C^∞ function. Then

(3.1) $[(X_1, X_2, ..., X_k),(Y_1, Y_2, ..., Y_k)](f_1, f_2, ..., f_k)$

$$=(X_1, X_2, ..., X_k)\{(Y_1, Y_2, ..., Y_k)(f_1, f_2, ..., f_k)\}-(Y_1, Y_2, ..., Y_k)$$

$$=[(X_1,Y_1]\ f_1, (X_2,Y_2]\ f_2, ...,(X_k,Y_k]\ f_k).$$

Suppose $K_i(X_i, Y_i, Z_i),\ i=1,2,....,k$ be the curvature tensors of the Hsu-structure manifolds $M_1, M_2, ..., M_k$ respectively. If $K(X,Y,Z)$ be the curvature tensor of the product manifold $M_1\times M_2\times....\times M_k$. Then we have

(3.2) $K(X,Y,Z)=[K_1(X_1,Y_1,Z_1), K_2(X_2,Y_2,Z_2), ..., K_k(X_k,Y_k,Z_k)].$

If $W=(W_1, W_2, ..., W_k)$ be a vector field on the product manifold, then

(3.3) $K'=(X,Y,Z,W)=g\,(K(X,Y,Z,W)),$

(3.4) $K'=(X,Y,Z,W)=K_1'(X_1,Y_1,Z_1,W_1)+K_2'(X_2,Y_2,Z_2,W_2)+....+K_k'(X_k,Y_k,Z_k,W_k)$

Thus, we have

Theorem 3.1: The product of manifold $M_1\times M_2\times....\times M_k$. is of constant curvature if and only if Hsu-structure manifolds $M_1, M_2, ..., M_k$ are separately of constant curvature.

Theorem 3.2: The Ricci tensor of the product manifold $M_1\times M_2\times....\times M_k$. is the sum of the Ricci tensor of the Hsu-structure manifolds $M_1, M_2, ..., M_k$

Theorem 3.3: The product of manifold $M_1\times M_2\times....\times M_k$. is an Einstein space if and only if the Hsu-structure manifolds $M_1, M_2, ..., M_k$ are separately Einstein space.

Proof: Let the product manifold $M_1\times M_2\times....\times M_k$. be an Einstein space. thus

(3.5) $Ric(X,Y)=Cg\ (X,Y)$,

where $\mathrm{C}=\frac{K}{n}$, K being the scalar curvature and n being the dimension of the product manifold. Then

$$Ric\ (X_i,Y_i)=Cg_i\ (X_i,Y_i), \qquad i=1,2,....,k.$$

Therefore the manifolds $M_1, M_2, ..., M_k$ are also Einstein spaces.

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